Shock attenuation of PMMA sandwich panels filled with soda-lime glass beads: A fluid-structure interaction continuum model simulation

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ABSTRACT

The dramatic increase of Improvised Explosive Device (IED) related injuries has stimulated many studies to reconsider the design of the current state-of-the-art vehicle and body protective equipment. Materials now need to be chosen not only to stop solid projectiles such as shrapnel or bullets but also to attenuate the injurious effects of incoming blast waves. New advanced computational models of such events have been proved to facilitate the access to information currently inaccessible to experiments. To this end, we developed a fluid–structure interaction computational continuum model to investigate the attenuation properties of foam specimens containing filler materials under shock loading. Three test specimens were examined: a solid foam sample, and two other foam samples incorporating an intermediate layer of filler material: SiO2 aerogel and soda-lime glass beads. The model was then calibrated and the results compared to the corresponding shock tube experimental results. In conclusion, the model shows good agreement with experiment values for the peak pressure of the transmitted wave as well as its propagation time. Complementing the existing experimental results, high density soda-lime glass beads filler material is shown to substantially decrease the peak magnitude of the transmitted wave and to decrease the spatial gradient of the pressure compared to the other lower density filler samples. However, the history of the sample reaction force suggests that the frame constraining the test specimen is being subjected to a higher impulse using the high density filler. Such a model paves the road to a new series of complex numerical models designed to accompany experimental testing by providing new insights on the mechanisms of fluid–structure interaction.

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1. Introduction

As the signature injury of recent military conflicts, Traumatic Brain Injuries (TBIs) caused by Improvised Explosive Device (IED) generated shock waves are attracting increasing medical and scientific attention [1–3]. In direct relation with these observations, the development of an advanced helmet liner able to provide enhanced protection against blast-induced TBI appears as a new necessary milestone.

The term blast wave refers to the pressure wave of finite amplitude that is generated by a rapid release of energy [4,5]. The four basic mechanisms of blast injury caused by such waves are characterized as: a) primary, when resulting from the direct impact of high pressure blast waves with body surfaces, b) secondary, when attributed to projectiles, debris and fragments, c) tertiary, when tissue injury results from impact with environmental structures such as buildings and d) quaternary, when the injuries result from heat, detonation products or electromagnetic pulses [5,6]. A significant percentage of blast-associated TBIs are due to more than one injury mechanism, often a combination of the blast component with acceleration–deceleration impact phenomena or fragment injuries. This combination has been referred to as a “blast-plus” injury [3].

The exact physical mechanisms by which blast waves cause TBIs are currently under investigation [1,3,7]. Several theories have been proposed to account for blast-induced TBIs, such as direct transmission of the blast wave to the brain [1] and propagation of the blast wave through the great vessels [7,8], orbital and aural openings, or even through the thorax, affecting the brain secondarily [1]. Studies have revealed that mechanisms of blast-induced trauma in solid tissues with varying density, such as the brain, can be correlated to wave metrics, such as impact pressure, velocity, wave duration and underpressure [1,7].

Complementing the underlying causes, symptoms, and injury patterns of TBI, substantial efforts have been directed toward the

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development of blast mitigation strategies. Proposed blast mitigation mechanisms include exploitation of material properties, such as porosity, inertial effects, thermal dissipation, and scattering effects [9–11]. The beneficial effect of Fluid–Structure Interaction (FSI) has also been well documented recently, and taken into account during the development of mitigation strategies [12–14].

Use of sandwich materials for protection against blast waves, consisting of two faces and a core, has been proposed in several research efforts [14–17]. The effect of increasing the mass fraction of the front face of the sandwich materials has also been examined [18]. Though this strategy increases the impulse required for complete crushing of the core material, it also undesirably increases the back face accelerations [18]. Studies on the scattering effects of stress waves in periodically layered composite materials show that, due to interface/microstructure scattering, the layered composites exhibit a larger shock front rise time [19]. Additionally, shock wave propagation is much slower than in either of its stand-alone components [19]. The effect of the multitude of interfaces in cellular media on stress wave propagation is examined in [20] using a one-dimensional spring-mass model, where results indicate attenuation of momentum and energy but possible increase of pressure levels. The effectiveness of soft condensed material, such as foams, and granular material, has also been tested in blast scenarios [10]. The study suggests that below a certain threshold of the material thickness relative to the charge size, increase of the pressure level rather than attenuation may occur [10]. In contrast to the previous findings, an extensive study of the attenuation capabilities of foams over a wide range of expansion ratios concludes that the entire range of density provides mitigation effects, with the optimal results corresponding to foams of 60:1 expansion ratio [21]. A similar experimental study suggests that low density materials, such as polystyrene foam or a mixture of 1:9 ratio of water to polystyrene, provide good levels of blast attenuation [22].

The purpose of this paper is the development of a numerical model aimed at simulating the response and the blast attenuation capabilities of foam helmet liners containing a variety of filler materials during shock tube experiments. The test samples that are evaluated and presented in this paper include a solid foam helmet liner sample that acts as the control sample and two foam liner specimens each containing an intermediate layer of filler material. Low density SiO2 aerogel and high density soda-lime glass spheres in a powder like form are used as the fillers. Shock tube experiments and numerical simulations have been employed to investigate the shock attenuation effectiveness of the foam specimens. The effectiveness of the liner specimens was evaluated with respect to: a) the peak transmitted pressure of an incoming blast wave, b) the spatial distribution of the transmitted wave energy and c) the time duration of the transmitted wave.

The proposed design of the helmet liner specimens stems from a previous design for sports applications utilizing fluid filled channels through the liner [23]. Experimental studies suggest that the use of a foam helmet liner with internal fluid filled channels and chambers offers increased protection against impacts [24,25]. Drop test experiments have indicated that the acceleration levels experienced with the use of a fluid filled helmet liner are significantly lower than the values attained using pure foam liners.

To test the effectiveness of the proposed helmet liner design, initial shock tube experiments were undertaken at Purdue University. The goal was to measure the blast attenuation capabilities of flat foam helmet liner samples containing a variety of filler materials. Test data indicate that the use of low density materials as fillers inside flat foam samples does not offer increased attenuation levels compared to the control sample constructed from pure foam of equal volume. On the other hand, significant attenuation has been observed when using solid high density materials and fluids [26–28]. A detailed description of the employed experimental setup, methods, and materials can be found in Ref. [26]. Both experiments and numerical computations show that the use of the soda-lime glass beads offers a substantial decrease in the peak transmitted pressure when compared to the control sample.

The simulation setup of the model is first presented in Section 2, while the constitutive models of the chosen materials are presented in Section 3. The simulation results are described in Section 4 and discussed in Section 5. A conclusion to the overall work is ultimately proposed in Section 6.

2. Simulation setup

A finite element model using the commercial software ABAQUS®/Explicit 6.10 [29] was developed for the numerical simulations. Simulations were run for a total of three cases: a solid foam specimen (control sample) and two samples incorporating an intermediate layer of filler material (SiO2 aerogel and soda-lime glass beads) inside the foam bulk. All samples were subjected to the same experimentally measured blast shock wave of 171 kPa (24.82 psi) overpressure measured in Ref. [26,28]. Note that this pressure level is higher than eardrum rupture level (∼35–103 kPa) but slightly lower than the reported lung damage threshold (∼200–517 kPa) [30].

In the following, we describe the experimental setup [26], its representation in the finite element model, and the loading condition.

2.1. Experimental setup

Test samples were sandwiched between two poly(methyl methacrylate) (PMMA) sheets and constrained by use of two horizontal 2.54 cm wide angle beams on two sides of the front face and four 2.54 cm wide angle beams on all sides of the rear face (see Fig. 1). The experimental stand included a PMMA chamber having a 19.05 cm × 19.05 cm square cross section placed behind the test sample. The rear end of the chamber was open to allow placement of the pressure transducer for the measurement of the transmitted wave profile 6.98 cm behind the rear surface of the sample along its centerline axis. The PMMA chamber isolates the pressure from external effects. A side view of the experimental setup is shown in Fig. 1 with the propagation direction of the incoming wave indicated by the red arrow.
2.2. Finite element model

The computational model contains two domains: one solid and one fluid. Modeling of both solid and fluid domains is achieved using a coupled Eulerian–Lagrangian (CEL) computational domain. The solid domain is discretized using a Lagrangian mesh and the fluid domains using an Eulerian formulation.

Two categories of sample configurations are tested; a solid foam specimen (control sample) and a foam sample with an internal cavity containing the investigated filler materials. The external dimensions of both configurations are 25.4 cm $\times$ 25.4 cm $\times$ 2.54 cm. An internal core of 20.32 cm $\times$ 25.4 cm $\times$ 1.27 cm is removed from the center of the solid foam specimen and is then occupied by the examined filler material. In compliance with the experimental setup, the foam samples are sandwiched between two PMMA sheets of 25.4 cm $\times$ 25.4 cm $\times$ 0.32 cm. Symmetry is exploited for computational efficiency and only a quarter of the foam samples and PMMA sheets are modeled. The PMMA sheets and the filler material location are depicted in the cross sectional view of a test sample shown in Fig. 2a. Appropriate symmetry boundary conditions have been imposed on the two sides of the solid domain corresponding to the symmetry planes. Assuming that the 2.54 cm wide angle beams of the experimental setup constrain the test samples perfectly, all degrees of freedom are constrained in the 2.54 cm wide regions of the PMMA delimited in red in Fig. 2b. The whole solid domain, including foam, PMMA and filler material regions, is modeled using 12,600 linear 8-node brick reduced integration solid continuum elements with enhanced hourglass control [31].

A close up view of the air domain is depicted in Fig. 3. Following the same symmetry condition as in the solid domain, only a quarter of the whole air column above and below the solid plate is modeled. The fluid domain is split into three primary sections labeled “Incoming”, “Specimen” and “Transmitted”. The Incoming section spans 50 cm above the top surface of the solid sample with a cross section of 10.3 cm $\times$ 12.7 cm. The external loading, simulating the incoming shock wave, is applied on its top surface. The shock front of the incoming wave after is reflected and propagates upstream toward the loading surface. Upon reaching the loading surface, it is observed that the reflected wave interacts with the loading surface of the fluid domain producing interference which then propagates back toward the specimen. The height of the Incoming Section of the fluid domain is thus selected such that the entire incoming shock profile propagates through the specimen before the aforementioned interference reaches the front surface of the specimen. The samples are placed inside the Specimen Section of the Eulerian domain. Finally, the Transmitted Section includes the column of air behind the foam samples inside the Plexiglas chamber present in the shock tube experiments. It is 1 m high and of 9.53 cm $\times$ 9.53 cm square cross section. The height of the Transmitted Section is determined such that the entire transmitted wave profile is recorded at the measurement location without any potential interference stemming from the interaction of the transmitted wave with the bottom surface of the Eulerian domain. Appropriate symmetry boundary conditions are applied on two sides of the fluid domain corresponding to the symmetry planes. A mesh convergence study has been conducted by increasing the mesh density of the Eulerian domain, with emphasis in the region near the solid specimen such as to accurately capture the FSI effects. During the mesh convergence investigation it was observed that numerical instabilities arise by increasing the mesh density above a threshold that corresponds to more than six elements across the solid specimen height. These instabilities originate from leakage effects observed in the fluid elements near the solid interface after the impingement of the shock wave and are due to the FSI numerical scheme used in ABAQUS®/Explicit 6.10. Therefore, the Eulerian fluid domain is discretized using 407,244 linear 8-node brick elements that correspond to the largest mesh density that does not initiate numerical instabilities, and is thus a good balance between accuracy and stability.

2.3. Shock loading condition

In the experiments, the test samples were placed perpendicular to the centerline axis of the shock tube 30.48 cm from its mouth. The shock wave profile was measured at the same distance but without the test sample. Because the wave dissipates as it propagates, the experimentally measured loading profile cannot be directly applied to the top surface of the Eulerian domain, 50 cm ahead of the test sample. To simulate the experimental loading, the measured incoming pressure profile is therefore modified based on a trial and error method and applied to the top surface of the domain such that at the level of the plate, the shock profile has the same magnitude and time duration as in the experiments with the plate removed.
Fig. 4 shows the pressure profile at the location of the front surface of the test samples for both experiment and simulation, and the initially applied pressure curve on the top of the fluid domain. Comparison between the experimental and numerical incoming shocks confirms that the peak overpressure is similar in both cases (\(\sim 171\) kPa), as is the negative phase (\(\sim -25\) kPa). The curves are similar enough to ensure that the loading in the simulation adequately matches the experimental curve.

3. Constitutive models

In this section, the material models for both solid and fluid materials are explained. The chosen constitutive laws for both materials are first presented, followed by the corresponding material properties.

3.1. Constitutive framework

3.1.1. Hugoniot/Mie-Grüneisen equation of state

The shock response of a wide range of materials is adequately described by the linear Hugoniot relation between the shock velocity \(U_s\), and particle velocity \(U_p\):\[ U_s = C_o + sU_p \] (1)

where \(C_o\) and \(s\) are experimentally obtained material parameters. The values of these parameters for many materials can be found in the literature [32,33].

Applying the principles of conservation of mass and momentum in a control volume at the shock front to Eq. (1), the pressure behind the shock front \(p_+\), can be expressed as [32]:\[ p_+ = \frac{\rho_o C_o^2 (1 - F_+)}{1 - s(1 - F_+)} \] (2)

where \(F_+\) is the deformation gradient immediately behind the shock front given by the following equation [32]:\[ F_+ = \frac{\rho_o}{\rho_+} = 1 + \frac{U_p}{U_s} \] (3)

where \(\rho_+\) is the material density behind the shock front.

Substituting Eq. (3) in Eq. (2) yields:

\[ p_+ = \rho_o U_s U_p \] (4)

Eq. (2) is also referred as the “Shock Hugoniot” and relates any final state of density to its corresponding pressure. Note that when the pressure is increased within the shock front the pressure does not actually move along the Hugoniot curve. Instead, pressure and density of the material change linearly in the \((p(1/\rho))\) plot from the initial state \((p_0(1/\rho_0))\) to the final shocked state \((p_+(1/\rho_+))\) along the “Rayleigh line”.

The linear Hugoniot model is commonly used in combination with the Mie–Grüneisen Equation of State (EOS) to describe the response of a material to shock loading [32,34]. The Mie–Grüneisen EOS relates the pressure \(p\), and internal energy \(E\) of a material to the final state of the Hugoniot [32]:\[ p = p_+ + \Gamma \rho (E - E_+) \] (5)

where \(\Gamma\) is the Grüneneisen coefficient, \(\rho\) the density and \(E_+\) the internal energy behind the shock.

The Grüneneisen coefficient \(\Gamma\) is given by:\[ \Gamma = \frac{\Gamma_o \rho_0}{\rho} \] (6)

where \(\Gamma_o\) is the Grüneneisen coefficient at the reference state.

Finally, substituting Eq. (6) into Eq. (5) yields:

\[ p = p_+ \left[ 1 - \frac{\Gamma_o (1 - F_+)}{2} \right] + \Gamma_o \rho_0 E \] (7)

In addition to using the Mie–Grüneisen/Hugoniot EOS for the volumetric behavior of the materials, a linear elastic model with a shear modulus \(G\), was employed in order to describe the deviatoric response.

3.1.2. Ideal Gas equation of state

Air was modeled using the ideal gas EOS:

\[ p + p_o = \rho RT \] (8)

where \(p_o\) is the atmospheric pressure, \(R\) the gas constant and \(T\) the absolute temperature.

In addition to the EOS, the specific energy \(E\), is given by Eq. (9), while the deviatoric stress \(S\), is calculated using Eq. (10):
3.2. Material properties

The foam used in this project, VN 600, is a closed cell foam based on a vinyl nitrile polymer acquired from DERTEX Corporation [35]. The VN600 foam was selected for its good energy absorption characteristics compared to other conventional foam types such as Expanded Polystyrene (EPS) and Polyurethane (PU) as demonstrated in previous drop test experiments [24 25]. Unfortunately, published data for the Dertex VN600 foam were not readily available. Therefore, the necessary material properties were calculated based on the approach described in detail in Appendix A.

Two materials were used as fillers: SiO2 aerogel and soda-lime glass beads. Aerogel is primarily used for thermal insulation and consists of approximately 95–99% air, the remaining material consisting of 1000–5000 μm size SiO2 particles [36]. Glass beads, a high density material in the form of an odorless white powder, consists of 250–420 μm size soda-lime glass spherical particles [27]. The parameters for both the Mie–Grüneisen/Hugoniot EOS and the shear modulus G were taken directly from published properties for PMMA [37] and SiO2 aerogel [38].

The granular soda-lime glass bead filler was modeled by an effective homogeneous material, the properties of which were determined through the following process. A one-dimensional column consisting of 37 spherical superimposed soda-lime glass beads of 350 μm diameter of total height 1.295 cm was modeled in ABAQUS®, see Fig. 5a. The material parameters of the individual glass beads were taken to be those of the bulk soda-lime glass material in the form of discs found in [39]. The first sphere of the column was subjected to a constant velocity loading condition corresponding to the particle velocity \( U_p \). The magnitude of \( U_p \) was calculated so as to simulate the experimental loading of 171 kPa.

Taking into account the bulk material properties of the soda-lime glass, the linear Hugoniot relation Eq. (1) and Eq. (4), the particle velocity was calculated to be \( U_p = 0.034 \text{ m/s} \). In order to calculate the material properties of the effective homogeneous material two simulations were conducted with \( U_p = 0.034 \text{ m/s} \) and \( U_p = 0.068 \text{ m/s} \) (thus doubling the velocity), see Fig. 5b. Through the reaction force, the transmitted pressure at the bottom sphere was calculated and finally, by use of Eq. (1) and Eq. (4), the \( C_p \) parameter of the linear Hugoniot relation for the effective homogeneous model was calculated. The value of the \( s \) parameter was taken to be zero since the magnitude of the particle velocity \( U_p \) was observed to be negligible compared to the calculated shock velocity \( U_p \). Finally, the shear modulus \( G \) of the homogeneous material model of the soda-lime glass beads was taken to be equal to that of the soda-lime glass bulk material as a first approximation, based on the observation that volumetric properties, and not deviatoric, are responsible for the transfer of pressure through the plate (note that this is confirmed in the sensibility analysis of Section 4.1).

Values for \( T_o \) were only found for PMMA \( (T_o = 0.75) \) [37], while no value was specified for the remaining materials. As a consequence, the response of these materials was assumed isothermal as a first approximation.

Table 1 shows the parameters for the Mie–Grüneisen/Hugoniot EOS and shear modulus for all the materials used in the numerical computations. Properties of bulk soda-lime glass and those corresponding to the effective homogeneous glass beads material are also included. Table 2 shows the material properties used for air.

### Table 1

<table>
<thead>
<tr>
<th>Material</th>
<th>( \rho_s ) [kg/m³]</th>
<th>( C_s ) [m/s]</th>
<th>( s )</th>
<th>( G ) [MPa]</th>
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<tr>
<td>Dertex VN600</td>
<td>108</td>
<td>108.44</td>
<td>1.35</td>
<td>0.405</td>
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<td>1.82</td>
<td>1148</td>
</tr>
<tr>
<td>Aerogel</td>
<td>128</td>
<td>567</td>
<td>1.08</td>
<td>4.17</td>
</tr>
<tr>
<td>Soda-lime glass bulk</td>
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<td>2010</td>
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<td>30400</td>
</tr>
<tr>
<td>Glass beads</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Homogeneous</td>
<td>2500</td>
<td>1860</td>
<td>0</td>
<td>30400</td>
</tr>
</tbody>
</table>

4. Results

In this section, the results concerning the distribution of the transmitted wave are first explored, followed by the evolution of
the reaction forces experienced by the frame, and the deflection of the samples.

4.1. Pressure distribution

The transmitted wave profile is measured for experiments and simulations at 6.98 cm from the rear surface of the sample along its centerline axis. The numerical solutions of the transmitted pressure for the control sample, and those using aerogel and glass beads as filler materials are shown in Fig. 6. A supplementary video that illustrates the numerical results of the shock wave propagation through the computational domain using the glass beads as filler material is provided as supporting data in the electronic version. Fig. 7 shows a comparison of the numerically calculated pressure profiles for the control sample and the two filler materials against the respective experimental measurements. The authors do not have multiple experimental curves for each tested configuration; however repeatability studies of the shock metrics of the incoming wave in Ref. [27] indicate that a maximum standard deviation of 15% was measured. This can be assumed as a reliable indication of the repeatability of the experiments. The pressure profiles for all data sets in Fig. 7 have been shifted in time such that \( t = 0 \) ms coincides with the moment the waves reaches the measurement location. Table 3 shows the values of a number of shock wave metrics for both experimentally and numerically determined transmitted wave profiles, where \( p_t \) is the peak transmitted pressure, \( t_a \) (arrival time) is the time required for the transmitted wave to propagate through the air and test sample and reach the measurement location, and \( t_r \) (rise time) is the time required to reach \( p_t \). Fig. 7c shows that the experimental transmitted pressure corresponding to the glass beads filler material is highly noisy, \textit{a priori} indicating higher standard deviation compared to the control and aerogel filler samples. The peak transmitted pressure \( p_t \) is determined as the average pressure value between 0.4 ms \( \leq t \leq 0.55 \) ms, while the rise time \( t_r \) lies between 0.4 and 0.5 ms. Table 4 contains the percentage error between numerical and experimental results based on the experimental values for the three shock wave metrics previously mentioned.

Supplementary video related to this article can be found at doi: 10.1016/j.ijimpeng.2012.03.003.

A sensitivity analysis regarding the properties used for the characterization of the Dertex VN 600 foam and PMMA has also been conducted in order to assess the influence of the material parameters on the response of the specimens. The use of a linear Hugoniot model, described in Section 3, was employed to characterize the volumetric response of the materials to shock loading. Examination of Eq. (4) shows that the pressure response of the material is a linear function of the shock velocity \( U_e \), which is also a linear function of the experimentally determined material parameters \( C_p \) and \( s \). Hence, the pressure response of the materials against shock loading is a linear function of \( C_p \) and \( s \) parameters. In other words, modifying either one of the parameters will modify both the shock velocity and the pressure response linearly. On the other hand, the effect of the shear modulus \( G \) on the material response is not known beforehand. The effect of the shear modulus on the blast attenuation effectiveness of the Dertex foam was examined by measuring the transmitted pressure in the case of the solid foam benchmark case by varying \( G \) to 80% and 120% of its original value. The experimentally measured transmitted pressure profile for the solid foam case and the numerical curves with the use of \( G \) at 80%, 100% and 120% of its original value are shown in Fig. 8, while the shock metrics are shown in Table 5. Similarly, the effect of the shear modulus of the PMMA sheets on the transmitted pressure profile of the shock wave was studied. The benchmark solid foam specimen sandwiched between two sheets of PMMA was employed, while modifying the \( G \) of the PMMA to 80% and 120% of its value. The transmitted pressure profiles are shown in Fig. 9 and the shock metrics in Table 6.

The results indicate that modifying the foam shear modulus by \( \pm 20\% \) has a negligible effect on the peak transmitted pressure (\(< 0.3\%) and the arrival time, while an increase of the shear modulus by 20% leads to a subsequent decrease of the rise time of the wave by approximately 3.5%. Similar trends are identified when modifying the shear modulus of the PMMA by \( \pm 20\% \); a 20% decrease of \( G \) results in a 2% increase of peak transmitted pressure, the change in arrival time is negligible while a 20% increase of \( G \) decreases the rise time by 3.5%. These results confirm that the transmitted pressure profile is relatively insensitive to the shear modulus of the foam and PMMA materials.

![Fig. 6. Numerical transmitted pressure profiles 6.98 cm from the rear surface of the sample along its centerline axis.](image-url)
In addition to the material properties sensitivity analysis, the effect of the employed artificial viscosity scheme on the calculated transmitted pressure wave profile was investigated. ABAQUS® employs two artificial viscosity coefficients so as to introduce damping associated with volumetric straining; linear and quadratic bulk viscosity. The linear and quadratic bulk viscosity parameters generate a bulk viscosity pressure equal to $p_{\text{bv1}} = b_1 \rho c_d L \varepsilon_{\text{vol}}$ and $p_{\text{bv2}} = \rho (b_2 L \varepsilon_{\text{vol}})^2$ respectively, where $b_1$ and $b_2$ are the damping coefficients, $\rho$ material density, $c_d$ dilatational wave speed, $L_e$ an element characteristic length and $\dot{\varepsilon}_{\text{vol}}$ the volumetric strain rate. The artificial viscosity parameters $b_1 = 0.2$ and $b_2 = 1.2$ were calibrated so as to obtain a good balance between the attenuation of high-frequency spurious modes and the minimization of the discontinuity width. Two simulations were undertaken based on the benchmark solid foam case; one with the linear artificial viscosity reduced to 50% of its value and a second where the quadratic viscosity term was also halved. The shock metric results of this investigation are shown in Table 7. Results show that reduction of the quadratic term by 50% of its original value has the largest effect on the peak pressure increasing the value by 3.3% compared to the benchmark case. The arrival time for all three cases maintains the same value, while the rise time decreases by 0.02 ms, suggesting a steeper pressure gradient. The final results prove to be relatively insensitive to the artificial viscosity parameters, and approximate values are thus sufficient.

The spatial distribution of the transmitted wave 6.98 cm behind the samples at the time when the transmitted pressure is maximum for each test sample is depicted in Fig. 10. The absolute pressure is used for the vertical axis in Fig. 10a, while normalized $p/p_{\text{max}}$ values are used in Fig. 10b. The centerline axis in both figures is noted with the dashed black line. Note that such plots not only allow for the three-dimensional identification of pressure values but also for the evaluation of the actual pressure spatial gradient within a given slice of material. The maximum pressure is experienced along the centerline axis, as anticipated; 5.585 kPa for the full foam control sample, 5.861 kPa for aerogel and 1.791 kPa for glass beads filler materials. The smallest values are recorded at the corner farthest from the centerline axis corresponding to the regions of the solid domain with the constrained translational degrees of freedom. The experienced pressure values for the aerogel case are always higher than the ones of the control sample, while the use of glass beads significantly reduces the value of the transmitted pressure by up to 47% compared to the lowest pressure value of the control sample. The use of the glass beads in

<table>
<thead>
<tr>
<th>Foam</th>
<th>Experimental</th>
<th>5999</th>
<th>0.35</th>
<th>0.13</th>
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<tr>
<td></td>
<td>Numerical</td>
<td>5585</td>
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<td></td>
<td>Numerical</td>
<td>1791</td>
<td>0.40</td>
<td>0.31</td>
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</table>

Table 3: Shock wave metrics for experimental and numerical transmitted wave profiles.

<table>
<thead>
<tr>
<th>Foam</th>
<th>$p_s$ [%]</th>
<th>$t_a$ [ms]</th>
<th>$t_r$ [ms]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foam</td>
<td>–6.89</td>
<td>8.57</td>
<td>123.07</td>
</tr>
<tr>
<td>Aerogel</td>
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<td>–14.29</td>
<td>–6.89</td>
</tr>
<tr>
<td>Glass beads</td>
<td>–6.96</td>
<td>8.11</td>
<td>–22.5–38</td>
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</table>

Table 4: Shock wave metrics error between experimental and numerical results.
addition to attenuating more effectively the incoming shock wave provides the most spatial uniform transmitted pressure profile as shown in Fig. 10b. The lowest transmitted value is approximately 13% less than the peak value for the glass beads case, while for the control sample and the aerogel, the respective values are 40% and 35%.

4.2. Reaction force and deflection evolution

The force that is applied to the frame constraining the plate impacted by the shock wave corresponds to the force that the head or neck of a person wearing the helmet liner would experience at equal area. Fig. 11 shows the overall reaction force measured at the constrained boundary condition regions (shown in Fig. 2b). The maximum values are 2.84 kN for the foam case, 3.18 kN for the glass beads and 3.17 kN for the aerogel filler. By integrating the reaction force over time up to the point when the reaction force reaches its first negative value, the impulse corresponding to the positive phase is calculated. The impulse for the control sample is 2.37 N·s, 2.44 N·s for the aerogel and 3.06 N·s for the glass beads. The foam sample thus exhibits the smallest peak reaction force and smallest impulse. The case with glass beads filler material exhibits a similar peak reaction force to the one for the aerogel filler; however the impulse is 25.4% higher.

Fig. 12 shows the numerically computed displacement of the point lying at the center of the rear PMMA sheet of the test sample along the direction of the shock propagation. The pure foam sample experiences the highest deflection with a value of 1.09 cm, while the glass beads filler sample exhibits the smallest deflection with less than 0.19 cm. The fact that the sample demonstrating the largest peak transmitted pressure reduction (glass beads) also exhibits the smallest vertical displacement indicates that the FSI effect in the shock attenuation is limited for this specific setup.

5. Discussion

The helmet liner attenuation of a shock wave was found to be superior for the glass beads filler, both experimentally and in the numerical simulations. However, the glass beads case presents further challenges in matching the encouraging experimental results.

All three experimental curves in Fig. 7 exhibit very high-frequency pressure fluctuations, a feature that is not present in the numerical profiles. Regardless of these fluctuations, the form of the pressure profile and the quantification of the shock wave metrics of Table 3 can be relatively well identified for the control sample and the aerogel filler case. This is not the case however for the glass beads filler. The large amplitude of the fluctuations relative to the small transmitted pressure values obscures the pressure profile of the transmitted wave. The experimental values for the peak pressure \( p_{\text{e}} \), and the rise time \( t_{\text{r}} \), for the glass beads case can only be approximated. The large fluctuations present in the glass beads case indicate significant dispersion of the shock wave during its propagation through the filler material. This stems from the granular nature of the glass beads, which provides a large number of interfaces that the shock wave encounters, and a larger distribution of contact forces between the beads [40].

Comparison between numerical and experimental shock wave metrics indicates that values of the peak pressure \( p_{\text{e}} \), and the arrival time of the transmitted wave \( t_{\text{e}} \), are in good agreement for all three test cases. The discrepancy between experimental and numerical values for \( p_{\text{e}} \) is less than 7% for all samples. The arrival time \( t_{\text{e}} \), for the aerogel filler case exhibits the largest error of approximately 14% while the error for the two remaining cases is approximately 8%. The largest discrepancies are found for the rise time \( t_{\text{r}} \), where the numerically computed value is roughly the same in all three simulations. The maximum discrepancy corresponds to the control sample at more than 120% of the experimental value. On the other hand, a single \( t_{\text{r}} \) value for the glass beads case cannot be accurately determined. Based on time span for the experimental value of \( t_{\text{r}} \), the numerical value is 22.5–38% lower than the experimental. The difference in \( t_{\text{r}} \) for the glass beads is attributed to the inability of the model to capture the dispersion effects that the shock wave experiences while propagating through materials with multiple interfaces [40]. In regard to the full foam sample, the computations suggest that the shock front is smoothed out, as compared to the measurement, resulting in an overestimation of

<table>
<thead>
<tr>
<th>Numerical</th>
<th>5599</th>
<th>0.38</th>
<th>0.30</th>
</tr>
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<tr>
<td>Numerical</td>
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<tr>
<td>Numerical</td>
<td>5601</td>
<td>0.38</td>
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</tr>
</tbody>
</table>

Table 5

Shock wave metrics for numerical transmitted wave profiles for control sample at 100%, 80% and 120% of Dertex VN 600 G.
the rise time $t_r$. This may be caused by the artificial viscosity present in the model.

When comparing experimental and numerical results, it should be taken into account that the exact laboratory conditions may not have been fully reproduced during the simulations. Discrepancies may have been introduced by a number of assumptions and simplifications that were made during the simulations, such as the assumption of a planar incoming wave and the firm containment of the solid samples by the 2.54 cm wide angle beams. Furthermore, the soda-lime glass beads were modeled by an effective homogeneous material after calculating the transmitted pressure through a column of glass beads vertically stacked. However, shock propagation through granular material is dependant on the topology of the grains since this influences both the density and local elastic stiffness [40]. Accordingly, positioning the individual beads in another configuration would affect the shock velocity $U_S$ and therefore the calculated parameters of the Hugoniot relation that were used for the effective homogeneous material. In addition to the vertical single stack of glass beads, an additional configuration was tested with the beads positioned in a 4 + 1 configuration (BCC type structure). This configuration was repeated until the total height of the stack reached 1.27 cm. The results show that the velocity of the transmitted wave is lower in the 4 + 1 configuration compared to the single stack case, indicating that the material properties of the effective homogeneous material provide and upper bound for the shock speed through the granular soda-lime glass beads. The complexity of the physics involved in wave propagation in granular media remains highly complex [40] and thus remains out of the scope of this paper. However, future works will aim at tackling this description by designing a more appropriate EOS.

Based on the sensitivity of the data acquisition system and the pressure transducers used during the experiments, an estimated experimental error of approximately ±700 Pa was calculated. The model proposed in this work thus provides good agreement with the experiment in regard to $p_t$ and $t_r$.

We finally complement our study with a spectral analysis of the transmitted wave profiles for experimental and numerical data for the three test specimens, see Fig. 13. The experimental transmitted wave profile of the foam specimen exhibits high intensity at frequencies of 1.52, 2.1, 2.75, 3.05 and 3.95 kHz. The numerical model identifies high intensity content at 1.550, 2.1 and 3.6 kHz. The experimental curve for the aerogel case exhibits high intensity at a single frequency of 1.52 kHz, while the numerical results indicate a peak at 2.03 kHz. In accordance with the previous discussion on glass bead filler modeling difficulties, the spectral study of this material lacks the discrete component of the real material and, apart from the first peak, the numerical model cannot capture the full experimental spectral behavior. The spectral analysis of the numerical and experimental results indicates that frequencies up to 4 kHz can be adequately captured by the numerical model. Note that such study is highly dependent on the geometry and boundary conditions of the specimen and would require new spectral studies for different configurations.

The results of this study show that the introduction of a high density material such as glass beads as an intermediate layer between two layers of foam reduces the transmitted pressure peak compared to a pure foam sample of the same volume by approximately 67%. Furthermore, the transmitted wave exhibits a lower spatial pressure gradient across the surface of the sample when using high density filler materials, an important feature if the requirement is not only the attenuation of the magnitude of a shock wave but also its spatial smoothening. Finally, with respect to the deflection of the samples, the results show that the displacement of the glass beads sample is 16.5% and 20.5% of the control and aerogel filler sample values respectively. The high peak pressure reduction in combination with the small deflection experienced by the glass beads filler suggests that the primary shock attenuation mechanism is the impedance mismatch between material interfaces while the

![Image](image.png)

**Fig. 9.** Experimental and numerical transmitted pressure profiles for control sample at 100%, 80% and 120% of PMMA shear modulus G.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$p_t$ [Pa]</th>
<th>$t_r$ [ms]</th>
<th>$t_f$ [ms]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical 80% G PMMA</td>
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</tr>
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<td>Numerical 100% G PMMA</td>
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<td>Numerical 120% G PMMA</td>
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<tr>
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</tbody>
</table>
Fig. 10. (a) Transmitted pressure 6.98 cm behind test sample; (b) Normalized transmitted pressure 6.98 cm behind test sample.

Fig. 11. Measured reaction forces of the constrained boundary regions.

Fig. 12. Displacement along the direction of the shock propagation at the bottom center point of solid domain.
contribution of FSI is limited. This study also examines the impulse and the reaction forces acting on the frame constraining the test samples which corresponds to the head and neck of the person wearing the helmet liner. The findings indicate that there is no clear correlation between the peak reaction force the frame experiences and the filler material density. However, the object that supports the test samples experiences a larger impulse when a high density filler is used, implying the need for a stronger support.

6. Conclusion

In this work, we proposed a fluid–structure interaction continuum framework aimed at accurately reproducing the attenuation properties of a set of chosen materials submitted to a shock wave. The numerical results presented here show a good correlation with the corresponding experimental values for both the peak pressure of the transmitted wave and the propagation time. The model not only confirms the higher shock attenuation ability of the glass bead filler material but also complements the experimental results by new numerical results usually inaccessible to experiments, such as pressure gradient spatial distribution or force/impulse evolutions. Such additional information is of drastic importance when considering the effect of a pressure wave on a supporting neck or head. As a conclusion, the proposed computational model constitutes the first step of a series of more complete models aimed at testing the shock attenuation effectiveness of various materials placed inside flat foam liner samples. By a priori identifying materials with superior blast attenuation capabilities (such as the proposed bead filler material) and making use of such models within a chosen set of metrics, further experiments or computations may be designed to accomplish the ultimate goal of designing a helmet liner for enhanced blast protection.

Acknowledgments

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Appendix A. VN 600 material properties

For a large number of materials, the $C_0$ parameter in the Hugoniot equation can be approximated with the bulk sound velocity $C_b$ [32], given by:

$$C_b = \sqrt{\frac{K}{\rho_0}} \quad (11)$$

where $K$ is the bulk modulus.

Uniaxial and hydrostatic compression tests were undertaken for the Dertex VN600 foam so as to determine experimental values for

![Fig. 13. (a) Experimental and numerical spectral analysis of transmitted wave profiles for control sample; (b) Aerogel; and (c) Glass beads filler material.](image-url)
the Young's modulus $E$, and bulk modulus $K$. Finally, assuming isotropic behavior, the shear modulus is calculated by:

$$G = \frac{3KE}{9K - E}$$ (12)

In order to obtain a value for the slope $s$ in Eq. (1), and considering the lack of experimental data for the VN600 foam, estimation was made based on published data for polyurethane foam (PU). PU foam was selected because of its similar nature to VN600 and its use in similar applications. The slopes were determined by selecting PU foam of $C_b$ value comparable to the respective value for VN 600 foam. Based on published data [41], shock wave propagation in PU foam of density $\rho_0 = 500 \text{ kg/m}^3$ is well described by the Hugoniot linear relation $U_s = 150 + 1.5U_p$, while for PU foam of density $\rho_0 = 320 \text{ kg/m}^3$, it can be described by $U_s = 100 + 1.32U_p$. Therefore, by linear interpolation, the slope of the equation relating shock to particle velocity was estimated to have the value of $s = 1.35$.

Table A-1 includes the experimentally derived material properties of $E$ and $K$, as well as the calculated values of $s$, $C_b$, and $G$ (using Eqs. (11) and (12)), for the Dertex VN600 foam.

<table>
<thead>
<tr>
<th>$\rho_0$ [kg/m$^3$]</th>
<th>$E$ [MPa]</th>
<th>$K$ [MPa]</th>
<th>$s$</th>
<th>$C_b$ [m/s]</th>
<th>$G$ [MPa]</th>
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<td>108.44</td>
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References


